Puissances

règles et définitions:

$$
\begin{aligned}
a^n &:= \underbrace{a \cdot \ldots \cdot a}_{n \text{ fois}} \\
a^m a^n &= a^{m+n} \\
(ab)^n &= a^n b^n \\
\left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\
a^{-n} &:= \frac{1}{a^n} \text{ (pour } a \neq 0) \\
\frac{a^n}{a^m} &= a^{n-m} \text{ (pour } a \neq 0) \\
(a^n)^m &= a^{nm} = (a^m)^n \\
a^0 &= 1 \text{ (pour } a \neq 0,\ 0^0 = 0,\ 0^n = 0) \\
a^{\frac{1}{m}} &:= \sqrt[m]{a} \text{ (pour } a \geq 0,\ m \neq 0) \\
a^{\frac{n}{m}} &= \left(a^{\frac{1}{m}}\right)^n = \left(\sqrt[m]{a}\right)^n \text{ (pour } a \geq 0,\ m \neq 0) \\
a^{\frac{n}{m}} &= (a^n)^{\frac{1}{m}} = \sqrt[m]{a^n} \text{ (pour } a \geq 0,\ m \neq 0)
\end{aligned}
$$

Calculer (exercices 1)

1. $5^{-3}5^6$
2. $4^3 4^{-5}$
3. $2^{-4}2^{-6}$
4. $\left(3^{-4}\right)^5$
5. $(3^4)^{-5}$
6. $(7^{-2})^{-3}$
7. $\frac{9^5}{9^5}$
8. $\frac{6^{-2}}{6^4}$
9. $\frac{5^{-5}}{5^{-2}}$

Calculer ou simplifier (exercices 2)

1. $(-1)^{12}$
2. $(-1)^{17}$
3. $(-3)^4$
4. $(-5)^3$
5. $(-a)^{24}$
6. $(-a)^{41}$
7. $(-2)^{-5}$
8. $\frac{1}{(-3)^{-2}}$
9. $4^2 4^3$
10. $7^{-2}7^5$
11. $5^5 5^{-4}$
12. $6(-6)^{-3}$
13. $(-3)^{-2}(-3)^{-3}$
14. $(-2)^3(-2)^5(-2)^{-4}$
15. $9^{-5}9^2 9^5$
16. $(-8)^{-5}(-8)^{17}(-8)^{-11}(-8)^{-1}$
17. $a^{2n}a^{-n}$
18. $(-a)^n(-a)^{2-n}$
19. $a^{n-2}a^{2n+1}$
20. $a^{2n-1}a^{1-n}a^{-n}$
21. $(2^3)^4$
22. $\left((-2)^{-3}\right)^2$
23. $(4^{-1})^{-3}$
24. $((-5)^3)^{-1}$
25. $\left((-a)^{-6}\right)^7$
26. $((-a)^5)^{-5}$
27. $\left((-a)^{-4}\right)^{-8}$
28. $(-2a)^{-4}$
29. $(a^2b^3)^5$
30. $(a^{-4}b^2)^{-3}$
31. $(a^5b^{-3})^{-1}$
32. $(a^{-3}b^{-4})^{-6}$
33. $\left(\frac{2}{5}\right)^3$
34. $\left(\frac{3}{2}\right)^{-4}$
35. $\left(-\frac{5}{3}\right)^{-2}$
36. $\left(\frac{4^5}{4^3}\right)$

37. $\frac{2^{-2}}{2^3}$
38. $\frac{7^{-8}}{7^{-9}}$
39. $\frac{(-a)^3}{(-a)^{-5}}$
40. $\frac{a^{2x-1}}{a^x}$
41. $\frac{(-a)^{x-1}}{(-a)^{1-x}}$
42. $\frac{(-a)^{x-1}}{(-a)^{x-2}}$

Calculer ou simplifier (exercices 3)

1. $(4^{-3})^2 16^2 \frac{1}{2^{-6}}$
2. $\frac{1}{(3^3)^2}\frac{1}{(27^{-2})^2}9^{-5}$
3. $\left(-\frac{5}{9}\right)^2\left(-\frac{3}{10}\right)^2$
4. $\left(\frac{a}{b}\right)^{-3}\left(\frac{b}{a}\right)^2$
5. $\left(\frac{2}{-5}\right)^4\left(\frac{-15}{16}\right)^2$
6. $\left(\frac{7^3}{4^2}\right)^{-4}\left(\frac{49^3}{25^2}\right)^2$
7. $\frac{4^{2t}}{2^t}$
8. $\frac{\left(\frac{2}{3}\right)^2\left(\frac{-2}{3}\right)^2}{\left(\left(\frac{-2}{3}\right)^4\right)^2}$
9. $\frac{\left(4^5\right)}{\left(\frac{1}{4}\right)^{-5}}$
10. $\frac{\left(\frac{4}{5}\right)^2}{\left(\frac{15}{4}\right)^{-1}}$
11. $\frac{\left(\frac{5\cdot 11}{3\cdot 4}\right)^{-3}}{\left(\frac{25\cdot 121}{9\cdot 16}\right)^{-2}}$
12. $\frac{\left(\frac{\frac{a}{b}}{\frac{b}{2}}\right)^{-2}}{\left(\frac{a^{-1}}{b^{-1}}\right)^2}$
13. $\frac{(3a)^{3x-1}}{(27a)^x}$
14. $\frac{\left(16a^3\right)^{2n}}{(8a^2)^{3n-1}}$

Calculer à l'aide des puissances de 10 (exercices 4)

1. $0.00005 \cdot 0.0006 \cdot 0.007$
2. $(30000)^2(2000000)^3$
3. $(4000000)^3(0.00002)^4$
4. $(0.0003)^4(0.001)^5$
5. $(62500)^4(0.000016)^4$

Simplifier (exercices 5)

1. $\sqrt{49a-147b}-\sqrt{7^2(a-3b)}$
2. $\left(5x\sqrt{\frac{2y}{50z^3}}\right)^2$
3. $\left(\frac{3x}{2y}\sqrt[3]{\frac{y^2}{x^2}}\right)^3$
4. $2a^2\sqrt{9a^2+81}+27a^2\sqrt{4a^2+36}$
5. $\sqrt{\frac{16a+32b}{25x-50y}}$
6. $\sqrt{\frac{4x^2}{9}-\frac{9y^2}{16}}$
7. $\frac{\sqrt{128a^2b^3}}{\sqrt{2b}}$
8. $\frac{\sqrt{125x^3y}}{\sqrt{5xy}}$
9. $\frac{\sqrt{\frac{50a}{27b}}}{\sqrt{\frac{2a^3}{3b^3}}}$
10. $\frac{\sqrt[3]{x^2+2xy+y^2}}{\sqrt[3]{x^2-y^2}}$
11. $\frac{\frac{1}{9}\sqrt{a}}{\frac{7}{10}\sqrt[3]{ab}}$
12. $\sqrt[a]{\sqrt{x^{2a}y^{6a}}}$
13. $\left(\frac{1}{27}\right)^{\frac{1}{3}}$
14. $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$
15. $\frac{4x^{\frac{3}{4}}}{5x^{-\frac{2}{3}}}$
16. $\sqrt[3]{x^{n+3}}+2x\sqrt[3]{x^n}$
17. $\sqrt[5]{x^3}\sqrt[5]{x^{y-2}}\sqrt[5]{x^{4-y}}$

Solutions

Solutions exercices 1

1. $5^{-3}5^6 = 125$
2. $4^3 4^{-5} = \frac{1}{16}$
3. $2^{-4}2^{-6} = \frac{1}{1024}$
4. $\left(3^{-4}\right)^5 = \frac{1}{3\,486\,784\,401} = 2.868\,0 \times 10^{-10}$

5. $(3^4)^{-5} = \frac{1}{3\,486\,784\,401} = 2.868\,0 \times 10^{-10}$
6. $(7^{-2})^{-3} = 117\,649$
7. $\frac{9^5}{9^5} = 1$
8. $\frac{6^{-2}}{6^4} = \frac{1}{46\,656} = 2.143\,3 \times 10^{-5}$
9. $\frac{5^{-5}}{5^{-2}} = \frac{1}{125} = 0.008$

Solutions exercices 2

1. $(-1)^{12} = 1$
2. $(-1)^{17} = -1$
3. $(-3)^4 = 81$
4. $(-5)^3 = -125$
5. $(-a)^{24} = a^{24}$
6. $(-a)^{41}$
7. $(-2)^{-5} = -\frac{1}{32} = -0.031\,25$
8. $\frac{1}{(-3)^{-2}} = 9$
9. $4^2 4^3 = 1024$
10. $7^{-2}7^5 = 343$
11. $5^5 5^{-4} = 5$
12. $6(-6)^{-3} = -\frac{1}{36} = -2.777\,8 \times 10^{-2}$
13. $(-3)^{-2}(-3)^{-3} = -\frac{1}{243} = -4.115\,2 \times 10^{-3}$
14. $(-2)^3(-2)^5(-2)^{-4} = 16$
15. $9^{-5}9^2 9^5 = 81$
16. $(-8)^{-5}(-8)^{17}(-8)^{-11}(-8)^{-1} = 1$
17. $a^{2n}a^{-n} = a^{2n-n} = a^{n(2-1)} = a^n$
18. $(-a)^n(-a)^{2-n} = a^2$
19. $a^{n-2}a^{2n+1} = a^{n-2+2n+1} = a^{3n-1}$
20. $a^{2n-1}a^{1-n}a^{-n} = a^{2n-1+1-n-n} = a$
21. $(2^3)^4 = 4096$
22. $\left((-2)^{-3}\right)^2 = \frac{1}{64}$
23. $(4^{-1})^{-3} = 64$
24. $((-5)^3)^{-1} = -\frac{1}{125}$
25. $\left((-a)^{-6}\right)^7 = \frac{1}{a^{42}} = a^{-42}$
26. $((-a)^5)^{-5} = -\frac{1}{a^{25}} = -a^{-25}$
27. $\left((-a)^{-4}\right)^{-8} = a^{32}$
28. $(-2a)^{-4} = \frac{1}{16a^4} = \frac{1}{16}a^{-4}$
29. $(a^2b^3)^5 = a^{10}b^{15}$
30. $(a^{-4}b^2)^{-3} = \frac{a^{12}}{b^6} = a^{12}b^{-6}$
31. $(a^5b^{-3})^{-1} = \frac{1}{a^5}b^3 = a^{-5}b^3$
32. $(a^{-3}b^{-4})^{-6} = a^{18}b^{24}$
33. $\left(\frac{2}{5}\right)^3 = \frac{8}{125} = 0.064$
34. $\left(\frac{3}{2}\right)^{-4} = \frac{16}{81} = 0.197\,53$
35. $\left(-\frac{5}{3}\right)^{-2} = \frac{9}{25} = 0.36$
36. $\left(\frac{4^5}{4^3}\right) = 16$
37. $\frac{2^{-2}}{2^3} = \frac{1}{32}$
38. $\frac{7^{-8}}{7^{-9}} = 7$
39. $\frac{(-a)^3}{(-a)^{-5}} = a^8$
40. $\frac{a^{2x-1}}{a^x} = a^{2x-1-x} = a^{x-1}$
41. $\frac{(-a)^{x-1}}{(-a)^{1-x}} = (-a)^{x-1-(1-x)} = (-a)^{2(x-1)}$
42. $\frac{(-a)^{x-1}}{(-a)^{x-2}} = (-a)^{x-1-(x-2)} = -a$

Solutions exercices 3

1. $(4^{-3})^2 16^2 \frac{1}{2^{-6}} = 4$
2. $\frac{1}{(3^3)^2}\frac{1}{(27^{-2})^2}9^{-5} = \frac{1}{81}$
3. $\left(-\frac{5}{9}\right)^2\left(-\frac{3}{10}\right)^2 = \frac{1}{36}$
4. $\left(\frac{a}{b}\right)^{-3}\left(\frac{b}{a}\right)^2 = \frac{1}{a^5}b^5 = a^{-5}b^5$
5. $\left(\frac{2}{-5}\right)^4\left(\frac{-15}{16}\right)^2 = \frac{9}{400} = 0.022\,5$
6. $\left(\frac{7^3}{4^2}\right)^{-4}\left(\frac{49^3}{25^2}\right)^2 = \frac{65\,536}{390\,625} = 0.167\,77$
7. $\frac{4^{2t}}{2^t} = 2^{4t-t} = 2^{3t}$

8. $\frac{\left(\frac{2}{3}\right)^2\left(\frac{-2}{3}\right)^2}{\left(\left(\frac{-2}{3}\right)^4\right)^2} = \frac{81}{16} = 5.0625$
9. $\frac{\left(4^5\right)}{\left(\frac{1}{4}\right)^{-5}} = 1$
10. $\frac{\left(\frac{4}{5}\right)^2}{\left(\frac{15}{4}\right)^{-1}} = \frac{12}{5} = 2.4$
11. $\frac{\left(\frac{5\cdot 11}{3\cdot 4}\right)^{-3}}{\left(\frac{25\cdot 121}{9\cdot 16}\right)^{-2}} = \frac{55}{12} = 4.5833$
12. $\frac{\left(\frac{a}{\frac{b}{2}}\right)^{-2}}{\left(\frac{a^{-1}}{b^{-1}}\right)^2} = \frac{1}{4}b^2$
13. $\frac{(3a)^{3x-1}}{(27a)^x} = \frac{(3a)^{3x-1}}{(3^3a)^x} = 3^{3x-1-3x}a^{3x-1-x} = 3^{-1}a^{2x-1}$
14. $\frac{\left(16a^3\right)^{2n}}{(8a^2)^{3n-1}} = \frac{\left(2^4a^3\right)^{2n}}{(3^3a^2)^{3n-1}} = \frac{2^{8n}a^{6n}}{3^{9n-3}a^{6n-2}} = 2^{8n-(9n-3)}a^{6n-(6n-2)} = 2^{-n+3}a^{-2}$

Solutions exercices 4

1. $0.00005\cdot 0.0006\cdot 0.007 = 5\cdot 10^{-5}\cdot 6\cdot 10^{-4}\cdot 7\cdot 10^{-3} = 210\cdot 10^{-12} = 2.1\cdot 10^2\cdot 10^{-12} = 2.1\cdot 10^{-10}$
2. $(30000)^2(2000000)^3 = \left(3\cdot 10^4\right)^2\cdot\left(2\cdot 10^6\right)^3 = 3^2 10^8 2^3 10^{18} = 9\cdot 8\cdot 10^{26} = 7.2\cdot 10^{27}$
3. $(4000000)^3(0.00002)^4 = 10.24$
4. $(0.0003)^4(0.001)^5 = 8.1\times 10^{-30}$
5. $(62500)^4(0.000016)^4 = 1$

Solutions exercices 5

1. $\sqrt{49a-147b} - \sqrt{7^2(a-3b)} = \sqrt{49a-147b} - \sqrt{49a-49\cdot 3b} = 0$
2. $\left(5x\sqrt{\frac{2y}{50z^3}}\right)^2 = 25x^2\frac{2y}{50z^3} = x^2\frac{2y}{2z^3} = x^2\frac{y}{z^3}$
3. $\left(\frac{3x}{2y}\sqrt[3]{\frac{y^2}{x^2}}\right)^3 = \frac{27x^3}{8y^3}\frac{y^2}{x^2} = \frac{27x}{8y}$
4. $2a^2\sqrt{9a^2+81} + 27a^2\sqrt{4a^2+36} = 2a^2\sqrt{9\left(a^2+9\right)} + 27a^2\sqrt{4\left(a^2+9\right)}$
   $= 2a^2 3\sqrt{(a^2+9)} + 27a^2 2\sqrt{(a^2+9)} = \sqrt{(a^2+9)}a^2\left(6+54\right) = 60\sqrt{(a^2+9)}a^2$
5. $\sqrt{\frac{16a+32b}{25x-50y}} = \sqrt{\frac{16(a+2b)}{25(x-2y)}} = \frac{4}{5}\sqrt{\frac{a+2b}{x-2y}}$
6. $\sqrt{\frac{4x^2}{9} - \frac{9y^2}{16}} = \sqrt{\frac{64x^2-81y^2}{144}} = \frac{1}{12}\sqrt{64x^2-81y^2} = \frac{1}{12}\sqrt{8^2x^2-9^2y^2}$
   $= \frac{1}{12}\sqrt{(8x)^2-(9y)^2} = \frac{1}{12}\sqrt{(8x-9y)(8x+9y)}$
7. $\frac{\sqrt{128a^2b^3}}{\sqrt{2b}} = \frac{8\sqrt{2}ab\sqrt{b}}{\sqrt{2}\sqrt{b}} = 8ab$
8. $\frac{\sqrt{125x^3y}}{\sqrt{5xy}} = \frac{5\sqrt{5}x\sqrt{x}\sqrt{y}}{\sqrt{5}\sqrt{x}\sqrt{y}} = 5x$
9. $\frac{\sqrt{\frac{50a}{27b}}}{\sqrt{\frac{2a^3}{3b^3}}} = \frac{\frac{5\sqrt{2}\sqrt{a}}{3\sqrt{3}\sqrt{b}}}{\frac{\sqrt{2}a\sqrt{a}}{\sqrt{3}b\sqrt{b}}} = \frac{5\sqrt{2}\sqrt{a}\sqrt{3}b\sqrt{b}}{3\sqrt{3}\sqrt{b}\sqrt{2}a\sqrt{a}} = \frac{5b}{3a}$
10. $\frac{\sqrt[3]{x^2+2xy+y^2}}{\sqrt[3]{x^2-y^2}} = \frac{\sqrt[3]{(x+y)^2}}{\sqrt[3]{(x-y)(x+y)}} = \sqrt[3]{\frac{(x+y)(x+y)}{(x-y)(x+y)}} = \sqrt[3]{\frac{(x+y)}{(x-y)}}$
11. $\frac{\frac{1}{9}\sqrt{a}}{\frac{7}{10}\sqrt[3]{ab}} = \frac{1\cdot 10}{9\cdot 7}\frac{a^{\frac{1}{2}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = \frac{10}{63}a^{\frac{1}{2}-\frac{1}{3}}b^{-\frac{1}{3}} = \frac{10}{63}a^{\frac{3}{6}-\frac{2}{6}}b^{-\frac{1}{3}} = \frac{10}{63}a^{\frac{1}{6}}b^{-\frac{1}{3}}$
12. $\sqrt[a]{\sqrt{x^{2a}y^{6a}}} = \left(\left(x^{2a}y^{6a}\right)^{\frac{1}{2}}\right)^{\frac{1}{a}} = \left(x^{2a}y^{6a}\right)^{\frac{1}{2a}} = x^{\frac{2a}{2a}}y^{\frac{6a}{2a}} = xy^3$

13. $\left(\frac{1}{27}\right)^{\frac{1}{3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

14. $\left(\frac{9}{4}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{9}{4}\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{9}{4}}} = \frac{1}{\frac{\sqrt{9}}{\sqrt{4}}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

15. $\frac{4x^{\frac{3}{4}}}{5x^{-\frac{2}{3}}} = \frac{4}{5}x^{\frac{3}{4}+\frac{2}{3}} = \frac{4}{5}x^{\frac{9+8}{12}} = \frac{4}{5}x^{\frac{17}{12}} = \frac{4}{5}x \cdot x^{\frac{5}{12}}$

16. $\sqrt[3]{x^{n+3}} + 2x\sqrt[3]{x^n} = x^{\frac{n+3}{3}} + 2x \cdot x^{\frac{n}{3}} = x^{\frac{n}{3}}x + 2x \cdot x^{\frac{n}{3}} = x^{\frac{n}{3}}x\left(1+2\right) = 3x^{\frac{n}{3}}x$

17. $\sqrt[5]{x^3}\sqrt[5]{x^{y-2}}\sqrt[5]{x^{4-y}} = x^{\frac{3}{5}}x^{\frac{y-2}{5}}x^{\frac{4-y}{5}} = x^{\frac{3+y-2+4-y}{5}} = x^{\frac{5}{5}} = x$