

# Potenzrechnen

Regeln und Definitionen:

$$a^n := \underbrace{a \cdot \dots \cdot a}_{n \text{ mal}}$$

$$a^m a^n = a^{m+n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} := \frac{1}{a^n} \text{ (für } a \neq 0\text{)}$$

$$\frac{a^n}{a^m} = a^{n-m} \text{ (für } a \neq 0\text{)}$$

$$(a^n)^m = a^{nm} = (a^m)^n$$

$$a^0 = 1 \text{ (für } a \neq 0, 0^0 = 0, 0^n = 0\text{)}$$

$$a^{\frac{1}{m}} := \sqrt[m]{a} \text{ (für } a \geq 0, m \neq 0\text{)}$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(\sqrt[m]{a}\right)^n \text{ (für } a \geq 0, m \neq 0\text{)}$$

$$a^{\frac{n}{m}} = (a^n)^{\frac{1}{m}} = \sqrt[m]{a^n} \text{ (für } a \geq 0, m \neq 0\text{)}$$

Berechnen Sie (Übungsblock 1)

- |                   |                    |                            |
|-------------------|--------------------|----------------------------|
| 1. $5^{-3}5^6$    | 4. $(3^{-4})^5$    | 7. $\frac{9^5}{9^5}$       |
| 2. $4^34^{-5}$    | 5. $(3^4)^{-5}$    | 8. $\frac{6^{-2}}{6^4}$    |
| 3. $2^{-4}2^{-6}$ | 6. $(7^{-2})^{-3}$ | 9. $\frac{5^{-5}}{5^{-2}}$ |

Berechnen oder vereinfachen Sie (Übungsblock 2)

- |                          |   |                                      |
|--------------------------|---|--------------------------------------|
| 1. $(-1)^{12}$           | 13. $(-3)^{-2}(-3)^{-3}$                    | 25. $((-a)^{-6})^7$                  |
| 2. $(-1)^{17}$           | 14. $(-2)^3(-2)^5(-2)^{-4}$                 | 26. $((-a)^5)^{-5}$                  |
| 3. $(-3)^4$              | 15. $9^{-5}9^29^5$                          | 27. $((-a)^{-4})^{-8}$               |
| 4. $(-5)^3$              | 16. $(-8)^{-5}(-8)^{17}(-8)^{-11}(-8)^{-1}$ | 28. $(-2a)^{-4}$                     |
| 5. $(-a)^{24}$           | 17. $a^{2n}a^{-n}$                          | 29. $(a^2b^3)^5$                     |
| 6. $(-a)^{41}$           | 18. $(-a)^n(-a)^{2-n}$                      | 30. $(a^{-4}b^2)^{-3}$               |
| 7. $(-2)^{-5}$           | 19. $a^{n-2}a^{2n+1}$                       | 31. $(a^5b^{-3})^{-1}$               |
| 8. $\frac{1}{(-3)^{-2}}$ | 20. $a^{2n-1}a^{1-n}a^{-n}$                 | 32. $(a^{-3}b^{-4})^{-6}$            |
| 9. $4^24^3$              | 21. $(2^3)^4$                               | 33. $\left(\frac{2}{5}\right)^3$     |
| 10. $7^{-2}7^5$          | 22. $((-2)^{-3})^2$                         | 34. $\left(\frac{3}{2}\right)^{-4}$  |
| 11. $5^55^{-4}$          | 23. $(4^{-1})^{-3}$                         | 35. $\left(-\frac{5}{3}\right)^{-2}$ |
| 12. $6(-6)^{-3}$         | 24. $((-5)^3)^{-1}$                         | 36. $\left(\frac{4^5}{4^3}\right)$   |

37.  $\frac{2^{-2}}{2^3}$

39.  $\frac{(-a)^3}{(-a)^{-5}}$

41.  $\frac{(-a)^{x-1}}{(-a)^{1-x}}$

38.  $\frac{7^{-8}}{7^{-9}}$

40.  $\frac{a^{2x-1}}{a^x}$

42.  $\frac{(-a)^{x-1}}{(-a)^{x-2}}$

**Berechnen oder vereinfachen Sie (Übungsblock 3)**

1.  $(4^{-3})^2 16^2 \frac{1}{2^{-6}}$

7.  $\frac{4^{2t}}{2^t}$

11.  $\frac{\left(\frac{5-11}{3-4}\right)^{-3}}{\left(\frac{25-121}{9-16}\right)^{-2}}$

2.  $\frac{1}{(3^3)^2} \frac{1}{(27^{-2})^2} 9^{-5}$

8.  $\frac{\left(\frac{2}{3}\right)^2 \left(\frac{-2}{3}\right)^2}{\left(\left(\frac{-2}{3}\right)^4\right)^2}$

12.  $\frac{\left(\frac{a}{b}\right)^{-2}}{\left(\frac{a-1}{b-1}\right)^2}$

3.  $\left(-\frac{5}{9}\right)^2 \left(-\frac{3}{10}\right)^2$

9.  $\frac{(4^5)}{\left(\frac{1}{4}\right)^{-5}}$

13.  $\frac{(3a)^{3x-1}}{(27a)^x}$

4.  $\left(\frac{a}{b}\right)^{-3} \left(\frac{b}{a}\right)^2$

5.  $\left(\frac{2}{-5}\right)^4 \left(\frac{-15}{16}\right)^2$

6.  $\left(\frac{7^3}{4^2}\right)^{-4} \left(\frac{49^3}{25^2}\right)^2$

10.  $\frac{\left(\frac{4}{5}\right)^2}{\left(\frac{15}{4}\right)^{-1}}$

14.  $\frac{(16a^3)^{2n}}{(8a^2)^{3n-1}}$

**Berechnen Sie mit Hilfe von 10er Potenzen (Übungsblock 4)**

1.  $0.00005 \cdot 0.0006 \cdot 0.007$

3.  $(4000000)^3 (0.00002)^4$

5.  $(62500)^4 (0.000016)^4$

2.  $(30000)^2 (2000000)^3$

4.  $(0.0003)^4 (0.001)^5$

**Vereinfachen Sie (Übungsblock 5)**

1.  $\sqrt{49a - 147b} - \sqrt{7^2(a - 3b)}$

9.  $\frac{\sqrt{\frac{50a}{27b}}}{\sqrt{\frac{2a^3}{3b^3}}}$

2.  $\left(5x \sqrt{\frac{2y}{50z^3}}\right)^2$

10.  $\frac{\sqrt[3]{x^2 + 2xy + y^2}}{\sqrt[3]{x^2 - y^2}}$

3.  $\left(\frac{3x}{2y} \sqrt[3]{\frac{y^2}{x^2}}\right)^3$

11.  $\frac{\frac{1}{9}\sqrt{a}}{\frac{7}{10}\sqrt[3]{ab}}$

4.  $2a^2\sqrt{9a^2 + 81} + 27a^2\sqrt{4a^2 + 36}$

12.  $\sqrt[3]{\sqrt{x^{2a}y^{6a}}}$

5.  $\sqrt{\frac{16a+32b}{25x-50y}}$

13.  $\left(\frac{1}{27}\right)^{\frac{1}{3}}$

6.  $\sqrt{\frac{4x^2}{9} - \frac{9y^2}{16}}$

14.  $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$

7.  $\frac{\sqrt{128a^2b^3}}{\sqrt{2b}}$

15.  $\frac{4x^{\frac{3}{4}}}{5x^{-\frac{2}{3}}}$

8.  $\frac{\sqrt{125x^3y}}{\sqrt{5xy}}$

16.  $\sqrt[3]{x^{n+3}} + 2x\sqrt[3]{x^n}$

17.  $\sqrt[5]{x^3} \sqrt[5]{xy^{-2}} \sqrt[5]{x^4-y}$

**Lösungen****Lösungen Übungsblock 1**

1.  $5^{-3}5^6 = 125$

3.  $2^{-4}2^{-6} = \frac{1}{1024}$

2.  $4^34^{-5} = \frac{1}{16}$

4.  $(3^{-4})^5 = \frac{1}{3486784401} = 2.8680 \times 10^{-10}$

5.  $(3^4)^{-5} = \frac{1}{348678401} = 2.8680 \times 10^{-10}$

6.  $(7^{-2})^{-3} = 117649$

7.  $\frac{9^5}{9^5} = 1$

8.  $\frac{6^{-2}}{6^4} = \frac{1}{46656} = 2.1433 \times 10^{-5}$

9.  $\frac{5^{-5}}{5^{-2}} = \frac{1}{125} = 0.008$

**Lösungen Übungsblock 2**

1.  $(-1)^{12} = 1$

2.  $(-1)^{17} = -1$

3.  $(-3)^4 = 81$

4.  $(-5)^3 = -125$

5.  $(-a)^{24} = a^{24}$

6.  $(-a)^{41}$

7.  $(-2)^{-5} = -\frac{1}{32} = -0.03125$

8.  $\frac{1}{(-3)^{-2}} = 9$

9.  $4^2 4^3 = 1024$

10.  $7^{-2} 7^5 = 343$

11.  $5^5 5^{-4} = 5$

12.  $6(-6)^{-3} = -\frac{1}{36} = -2.7778 \times 10^{-2}$

13.  $(-3)^{-2} (-3)^{-3} = -\frac{1}{243} = -4.1152 \times 10^{-3}$

14.  $(-2)^3 (-2)^5 (-2)^{-4} = 16$

15.  $9^{-5} 9^2 9^5 = 81$

16.  $(-8)^{-5} (-8)^{17} (-8)^{-11} (-8)^{-1} = 1$

17.  $a^{2n} a^{-n} = a^{2n-n} = a^{n(2-1)} = a^n$

18.  $(-a)^n (-a)^{2-n} = a^2$

19.  $a^{n-2} a^{2n+1} = a^{n-2+2n+1} = a^{3n-1}$

20.  $a^{2n-1} a^{1-n} a^{-n} = a^{2n-1+1-n-n} = a$

21.  $(2^3)^4 = 4096$

22.  $((-2)^{-3})^2 = \frac{1}{64}$

23.  $(4^{-1})^{-3} = 64$

24.  $((-5)^3)^{-1} = -\frac{1}{125}$

25.  $((-a)^{-6})^7 = \frac{1}{a^{42}} = a^{-42}$

26.  $((-a)^5)^{-5} = -\frac{1}{a^{25}} = -a^{-25}$

27.  $((-a)^{-4})^{-8} = a^{32}$

28.  $(-2a)^{-4} = \frac{1}{16a^4} = \frac{1}{16} a^{-4}$

29.  $(a^2 b^3)^5 = a^{10} b^{15}$

30.  $(a^{-4} b^2)^{-3} = \frac{a^{12}}{b^6} = a^{12} b^{-6}$

31.  $(a^5 b^{-3})^{-1} = \frac{1}{a^5} b^3 = a^{-5} b^3$

32.  $(a^{-3} b^{-4})^{-6} = a^{18} b^{24}$

33.  $(\frac{2}{5})^3 = \frac{8}{125} = 0.064$

34.  $(\frac{3}{2})^{-4} = \frac{16}{81} = 0.19753$

35.  $(-\frac{5}{3})^{-2} = \frac{9}{25} = 0.36$

36.  $(\frac{4^5}{4^3}) = 16$

37.  $\frac{2^{-2}}{2^3} = \frac{1}{32}$

38.  $\frac{7^{-8}}{7^{-9}} = 7$

39.  $\frac{(-a)^3}{(-a)^{-5}} = a^8$

40.  $\frac{a^{2x-1}}{a^x} = a^{2x-1-x} = a^{x-1}$

41.  $\frac{(-a)^{x-1}}{(-a)^{1-x}} = (-a)^{x-1-(1-x)} = (-a)^{2(x-1)}$

42.  $\frac{(-a)^{x-1}}{(-a)^{x-2}} = (-a)^{x-1-(x-2)} = -a$

**Lösungen Übungsblock 3**

1.  $(4^{-3})^2 16^2 \frac{1}{2^{-8}} = 4$

2.  $\frac{1}{(3^3)^2} \frac{1}{(27^{-2})^2} 9^{-5} = \frac{1}{81}$

3.  $(-\frac{5}{9})^2 (-\frac{3}{10})^2 = \frac{1}{36}$

4.  $(\frac{a}{b})^{-3} (\frac{b}{a})^2 = \frac{1}{a^5} b^5 = a^{-5} b^5$

5.  $(\frac{2}{-5})^4 (\frac{-15}{16})^2 = \frac{9}{400} = 0.0225$

6.  $(\frac{7^3}{4^2})^{-4} (\frac{49^3}{25^2})^2 = \frac{65536}{390625} = 0.16777$

7.  $\frac{4^{2t}}{2^t} = 2^{4t-t} = 2^{3t}$

8.  $\frac{\left(\frac{2}{3}\right)^2 \left(\frac{-2}{3}\right)^2}{\left(\left(\frac{-2}{3}\right)^4\right)^2} = \frac{81}{16} = 5.0625$
9.  $\frac{(4^5)}{\left(\frac{1}{4}\right)^{-5}} = 1$
10.  $\frac{\left(\frac{4}{5}\right)^2}{\left(\frac{15}{4}\right)^{-1}} = \frac{12}{5} = 2.4$
11.  $\frac{\left(\frac{5 \cdot 11}{3 \cdot 4}\right)^{-3}}{\left(\frac{25 \cdot 121}{9 \cdot 16}\right)^{-2}} = \frac{55}{12} = 4.5833$
12.  $\frac{\left(\frac{\frac{a}{b}}{\frac{b}{a}}\right)^{-2}}{\left(\frac{a-1}{b-1}\right)^2} = \frac{1}{4}b^2$
13.  $\frac{(3a)^{3x-1}}{(27a)^x} = \frac{(3a)^{3x-1}}{(3^3a)^x} = 3^{3x-1-3x} a^{3x-1-x} = 3^{-1} a^{2x-1}$
14.  $\frac{(16a^3)^{2n}}{(8a^2)^{3n-1}} = \frac{(2^4a^3)^{2n}}{(3^3a^2)^{3n-1}} = \frac{2^{8n} a^{6n}}{3^{9n-3} a^{6n-2}} = 2^{8n-(9n-3)} a^{6n-(6n-2)} = 2^{-n+3} a^{-2}$

#### Lösungen Übungsblock 4

1.  $0.00005 \cdot 0.0006 \cdot 0.007 = 5 \cdot 10^{-5} \cdot 6 \cdot 10^{-4} \cdot 7 \cdot 10^{-3} = 210 \cdot 10^{-12} = 2.1 \cdot 10^2 \cdot 10^{-12} = 2.1 \cdot 10^{-10}$
2.  $(30000)^2 (2000000)^3 = (3 \cdot 10^4)^2 \cdot (2 \cdot 10^6)^3 = 3^2 10^8 2^3 10^{18} = 9 \cdot 8 \cdot 10^{26} = 7.2 \cdot 10^{27}$
3.  $(4000000)^3 (0.00002)^4 = 10.24$
4.  $(0.0003)^4 (0.001)^5 = 8.1 \times 10^{-30}$
5.  $(62500)^4 (0.000016)^4 = 1$

#### Lösungen Übungsblock 5

1.  $\sqrt{49a-147b} - \sqrt{7^2(a-3b)} = \sqrt{49a-147b} - \sqrt{49a-49 \cdot 3b} = 0$
2.  $\left(5x\sqrt{\frac{2y}{50z^3}}\right)^2 = 25x^2 \frac{2y}{50z^3} = x^2 \frac{2y}{z^3} = x^2 \frac{y}{z^3}$
3.  $\left(\frac{3x}{2y} \sqrt{\frac{y^2}{x^2}}\right)^3 = \frac{27x^3}{8y^3} \frac{y^2}{x^2} = \frac{27x}{8y}$
4.  $2a^2\sqrt{9a^2+81} + 27a^2\sqrt{4a^2+36} = 2a^2\sqrt{9(a^2+9)} + 27a^2\sqrt{4(a^2+9)}$   
 $= 2a^2 \cdot 3\sqrt{(a^2+9)} + 27a^2 \cdot 2\sqrt{(a^2+9)} = \sqrt{(a^2+9)} a^2 (6+54) = 60\sqrt{(a^2+9)} a^2$
5.  $\sqrt{\frac{16a+32b}{25x-50y}} = \sqrt{\frac{16(a+2b)}{25(x-2y)}} = \frac{4}{5} \sqrt{\frac{a+2b}{x-2y}}$
6.  $\sqrt{\frac{4x^2}{9} - \frac{9y^2}{16}} = \sqrt{\frac{64x^2-81y^2}{144}} = \frac{1}{12} \sqrt{64x^2-81y^2} = \frac{1}{12} \sqrt{8^2x^2-9^2y^2}$   
 $= \frac{1}{12} \sqrt{(8x)^2 - (9y)^2} = \frac{1}{12} \sqrt{(8x-9y)(8x+9y)}$
7.  $\frac{\sqrt{128a^2b^3}}{\sqrt{2b}} = \frac{8\sqrt{2ab}\sqrt{b}}{\sqrt{2}\sqrt{b}} = 8ab$
8.  $\frac{\sqrt{125x^3y}}{\sqrt{5xy}} = \frac{5\sqrt{5x}\sqrt{x}\sqrt{y}}{\sqrt{5}\sqrt{x}\sqrt{y}} = 5x$
9.  $\frac{\sqrt{\frac{50a}{27b}}}{\sqrt{\frac{2a^3}{3b^3}}} = \frac{\frac{5\sqrt{2}\sqrt{a}}{3\sqrt{3}\sqrt{b}}}{\frac{\sqrt{2a}\sqrt{a}}{\sqrt{3b}\sqrt{b}}} = \frac{5\sqrt{2}\sqrt{a}\sqrt{3b}\sqrt{b}}{3\sqrt{3}\sqrt{b}\sqrt{2a}\sqrt{a}} = \frac{5b}{3a}$
10.  $\frac{\sqrt[3]{x^2+2xy+y^2}}{\sqrt[3]{x^2-y^2}} = \frac{\sqrt[3]{(x+y)^2}}{\sqrt[3]{(x-y)(x+y)}} = \sqrt[3]{\frac{(x+y)(x+y)}{(x-y)(x+y)}} = \sqrt[3]{\frac{(x+y)}{(x-y)}}$
11.  $\frac{\frac{1}{9}\sqrt{a}}{\frac{7}{10}\sqrt[3]{ab}} = \frac{1 \cdot 10}{9 \cdot 7} \frac{a^{\frac{1}{2}}}{a^{\frac{1}{3}} b^{\frac{1}{3}}} = \frac{10}{63} a^{\frac{1}{2}-\frac{1}{3}} b^{-\frac{1}{3}} = \frac{10}{63} a^{\frac{3}{6}-\frac{2}{6}} b^{-\frac{1}{3}} = \frac{10}{63} a^{\frac{1}{6}} b^{-\frac{1}{3}}$
12.  $\sqrt[a]{\sqrt{x^2ay^6a}} = \left(\left(x^2ay^6a\right)^{\frac{1}{2}}\right)^{\frac{1}{a}} = \left(x^2ay^6a\right)^{\frac{1}{2a}} = x^{\frac{2a}{2a}} y^{\frac{6a}{2a}} = xy^3$

$$13. \left(\frac{1}{27}\right)^{\frac{1}{3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$$

$$14. \left(\frac{9}{4}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{9}{4}\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{9}{4}}} = \frac{1}{\frac{\sqrt{9}}{\sqrt{4}}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$15. \frac{4x^{\frac{3}{4}}}{5x^{-\frac{2}{3}}} = \frac{4}{5}x^{\frac{3}{4}+\frac{2}{3}} = \frac{4}{5}x^{\frac{9+8}{12}} = \frac{4}{5}x^{\frac{17}{12}} = \frac{4}{5}x \cdot x^{\frac{5}{12}}$$

$$16. \sqrt[3]{x^{n+3}} + 2x\sqrt[3]{x^n} = x^{\frac{n+3}{3}} + 2x \cdot x^{\frac{n}{3}} = x^{\frac{n}{3}}x + 2x \cdot x^{\frac{n}{3}} = x^{\frac{n}{3}}x(1+2) = 3x^{\frac{n}{3}}x$$

$$17. \sqrt[5]{x^3}\sqrt[5]{xy^{-2}}\sqrt[5]{x^4y} = x^{\frac{3}{5}}x^{\frac{y-2}{5}}x^{\frac{4-y}{5}} = x^{\frac{3+y-2+4-y}{5}} = x^{\frac{5}{5}} = x$$